Elementary Particle Physics

from the context of the courses
PHY 493: Elementary Particle Physics

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Contents

0.1 The SI System

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed bellow:

Type	Unit	Definition
Length	Meter(m)	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram(kg)	Defined by fixing the Planck's constant $h = 6.62607015 \times 10^{-34} kg\ m^2 s^{-1}$
Time	Second(s)	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere(A)	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin(K)	Defined by fixing the value of the Boltzmann constant k to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed bellow:

Prefix	Symbol	Definition
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Additionally, the following are defined constants:

Symbol	Definition
$\hbar$	$\hbar = \frac{h}{2\pi} \approx 1.0546 \times 10^{-34} kg\ m^2 s^{-1}$

Chapter 1

Fundamental Particles

1.1 Fermions and Bosons

Definition 1.1.1. A **fermion** is a particle with half integer spin.

Definition 1.1.2. The **color** of a particle is a quantum number that can be in 7 possibles states: colorless, red, green, blue, anti-red, anti-green, and anti-blue.

Definition 1.1.3. A **quark** is a fermion with color charge and an **anti-quark** is a fermion with anti color charge.

Table 1.1.4. Quarks and Anti-Quarks Table of quarks and anti-quarks and there corresponding properties.

Name	Sym.	S	Q	B_a	T_3	I_3	C	S	T	B_o	Mass (MeV/c^2)
Up	u	1/2	2/3	1/3	1/2	1/2					2.3
Anti-Up	$\overline{u}$	1/2	$-2/3$	$-1/3$	$-1/2$	$-1/2$					2.3
Down	d	1/2	$-1/3$	1/3	$-1/2$	$-1/2$					4.8
Anti-Down	$\overline{d}$	1/2	1/3	$-1/3$	1/2	1/2					4.8
Charm	c	1/2	2/3	1/3	1/2		1				1.275×10^3
Anti-Charm	$\bar{c}$	1/2	$-2/3$	$-1/3$	$-1/2$		-1				1.275×10^3
Strange	s	1/2	$-1/3$	1/3	$-1/2$			-1			1.01×10^2
Anti-Strange	$\bar{s}$	1/2	1/3	$-1/3$	1/2			1			1.01×10^2
Top	t	1/2	2/3	1/3	1/2				1		1.72×10^5
Anti-Top	$\bar{t}$	1/2	$-2/3$	$-1/3$	$-1/2$				-1		1.72×10^5
Bottom	b	1/2	$-1/3$	1/3	$-1/2$					-1	4.19×10^3
Anti-Bottom	$\overline{b}$	1/2	1/3	$-1/3$	1/2					1	4.19×10^3

S is spin ($\hbar$), Q is electric charge (e), B_a is baryon number, I_3 is strong isospin, T_3 is weak isospin, C is charmness, S is strangeness, T is topness, B_o is bottomness.

Definition 1.1.5. a **lepton** or an **anti-lepton** is a fermion with no color charge.

Table 1.1.6. Leptons and Anti-Leptons Table of leptons and anti-leptons and their corresponding properties.

Name	Sym.	S	Q	L_e	L_μ	L_τ	T_3	Mass (MeV/c^2)
Electron	e	1/2	-1	1			$-1/2$	5.10998×10^{-1}
Anti-Electron	$\bar{e}$	1/2	1	-1			1/2	5.10998×10^{-1}
Muon	μ	1/2	-1		1		$-1/2$	1.0565×10^2
Anti-Muon	$\overline{\mu}$	1/2	1		-1		1/2	1.0565×10^2
Tau	τ	1/2	-1			1	$-1/2$	1.776×10^3
Anti-Tau	$\overline{\tau}$	1/2	1			-1	1/2	1.776×10^3
Electron Neutrino	e	1/2		1			1/2	< 0.0000022
Electron Anti-Neutrino	$\bar{e}$	1/2		-1			$-1/2$	< 0.0000022
Muon Neutrino	ν_μ	1/2			1		1/2	< 0.17
Muon Anti-Neutrino	$\overline{\nu} - \mu$	1/2			-1		$-1/2$	< 0.17
Tau Neutrino	τ	1/2				1	1/2	< 15.5
Tau Anti-Neutrino	$\overline{\tau}$	1/2				-1	$-1/2$	< 15.5

S is spin ($\hbar$), Q is electric charge (e), L_e is lepton electron number, L_e is lepton electron number, L_μ is lepton muon number, L_τ is lepton tau number, and T_3 is weak isospin.

Definition 1.1.7. A **Boson** is a particle with integer spin.

Table 1.1.8. Bosons Table of bosons and their corresponding properties.

Name	Sym.	S	Q	T_3
Photon	γ	1		or 1
Positive Weak	W^+	1	1	1
Neutral Weak	Z^0	1		or 1
Negative Weak	W^-	1	-1	-1
Gluon	g	1		
Higgs	H^0			

S is spin ($\hbar$), Q is electric charge (e), and T_3 is weak isospin.

Chapter 2

Interactions

2.1 Forces

Definition 2.1.1. The **electromagnetic force** is the fundamental force of nature mediated by γ boson (photon).

Definition 2.1.2. The **weak force** is the fundamental force of nature mediated by W^+, Z^0, and W^- bosons.

Definition 2.1.3. The **strong force** is the fundamental force of nature mediated by g boson.

Theorem 2.1.4. Heisenburg's Uncertainty Principle states that the product of the uncertainty ΔE of energy and the uncertainty Δt of time is greater than $\hbar/2$.

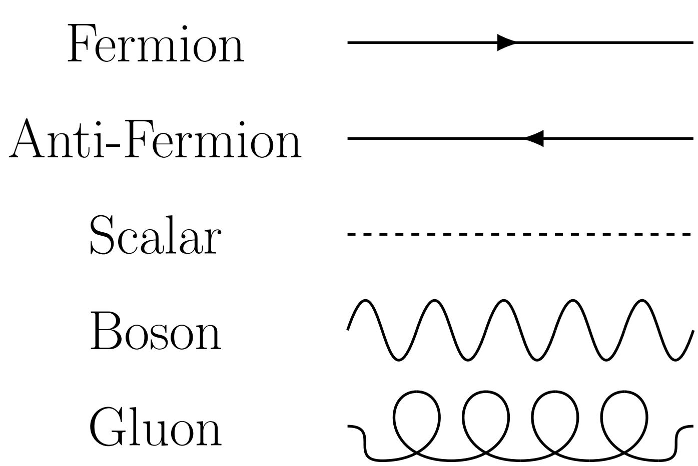
$$\Delta E \Delta t \geq \hbar/2 \tag{2.1.4}$$

Definition 2.1.5. The **range of a forces** is the maximum distance that a force can act determined by the uncertainty principle and the mass of the particles mediating the force.

$$r \leq \tau \times c = \frac{\hbar c}{2mc^2} \tag{2.1.5}$$

2.2 Feynman Diagrams

Definition 2.2.1. A **Feynman diagram** is a pictorial representation of and interaction between particles. Diagrams are drawn with lines representing particles connected points of interaction between those particles. The x-axis represents time and the y axis represents space.



2.3 Probability of Interaction

Definition 2.3.1. A **leading order interaction** is an interaction with a Feynman diagram with 2 vertices.

Definition 2.3.2. An **nth order interaction** is an interaction with a Feynman diagram with $2n$ vertices.

Definition 2.3.3. The **probability of interaction** denoted σ is the probability of an interaction.

Proposition 2.3.4. The probability of interaction is proportional to g^n where n is the order of the interaction.

$$\sigma \propto g^n \tag{2.3.4}$$

Proposition 2.3.5. The probability of interaction is proportional to $|M_{TOT}|^2$ where M_TOT is the total matrix probability.

$$M_{TOT} = M_1 + M_2 + \dots \tag{2.3.5}$$

Theorem 2.3.6. Particles can decay into other particles if the follow applies:

1. The particle must decay into a lower energy/mass final state.
2. The conservation rules must be held for the mediator of the decay.

Definition 2.3.7. The **decay constant** denoted Γ is the rate at which a particle decays where $N(t)$ is the number of particles at time t.

$$N(t) = N(0)e^{-\Gamma t} \tag{2.3.7}$$

Definition 2.3.8. The **natural life time** denoted τ is the reciprocal of the decay constant $\tau = 1/\Gamma$.

Theorem 2.3.9. The decay rate depends on the square of the matrix element M and the mass of the particle m.

$$\Gamma = \frac{1}{2m}|M|^2 \tag{2.3.9}$$

2.4 Rutherford Scattering

Definition 2.4.1. The **Rutherford scattering experiment** was an early experiment that demonstrated the approximate size of the nucleus. The scattering of electrons off a charged nucleus can be approximated with

$$N = \frac{N_i n L Z^2 k^2 e^4}{4r^2 E^2 \sin^4(\theta/2)} \tag{2.4.1}$$

where N is the rate of detected α particles, N_i is the rate of incoming α particles, n is atoms per unit volume, L is thickness of the target, Z is the atomic number, r is the distance from the target, E is the kinetic energy, and θ is the scattering angle.

Chapter 3

Relativity

3.1 4-Vector Coordinates

Definition 3.1.1. The **space-time coordinate** is a vector in $\mathbb{R}^4$ that represents a time and position in a reference frame.

$$(ct, x, y, z) \tag{3.1.1}$$

Definition 3.1.2. The **energy-momentum coordinate** is a vector in $\mathbb{R}^4$ that represents an energy and momentum in a reference frame.

$$(E/c^2, p_x, p_y, p_z) \tag{3.1.2}$$

Definition 3.1.3. The **Lorentz factor** denoted γ is the quantity expressing how much the time dilation and space contraction affects an object in a reference frame and β is the frame of the speed of light that an object is moving in a reference frame.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c} \tag{3.1.3}$$

Theorem 3.1.4. The **Lorentz transformation** is the transformation of space-time coordinates into a reference frame moving with velocity v in the x direction.

$$L = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.1.4}$$

Proposition 3.1.5. The effect on an object due to length contraction L' and time dilation t' is determined by the γ of the object in a reference frame.

$$L' = L_0/\gamma, \quad t' = \gamma t_0 \tag{3.1.5}$$

Proposition 3.1.6. The velocity sum of two objects at velocities v_A and a_B

Definition 3.1.7. The **position invariant** p and the **momentum invariant** s are defined as

$$p^2 = c^2t^2 - x^2 - y^2 - z^2 \tag{3.1.7}$$

$$s^2 = E^2/c^4 - p_x^2 - p_y^2 - p_z^2 \tag{3.1.7}$$

Proposition 3.1.8. The invariants s and p are the same for all reference frames.

3.2 Inner Product Space

Chapter 4

Symmetry

4.1 Angular Momentum

Chapter 5

Bound States

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

m_1 m_2	J M	J M	...
m_1 m_2	Coefficients		
⋮			

1/2×1/2

m_1 m_2	1 +1
+1/2 +1/2	1

m_1 m_2	1 0	0 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1 −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2 +3/2
+1 +1/2	1

m_1 m_2	3/2 +1/2	1/2 +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2 −1/2	1/2 −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2 −3/2
−1 −1/2	1

2×1/2

m_1 m_2	5/2 +5/2
+2 +1/2	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2 +1/2	3/2 +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2 −1/2	3/2 −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2 −3/2	3/2 −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2 −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2 +2
+3/2 +1/2	1

m_1 m_2	2 +1	1 +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2 0	1 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2 −1	1 −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2 −2
−3/2 −1/2	1

2×1

m_1 m_2	3 +3
+2 +1	1

m_1 m_2	3 +2	2 +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3 +1	2 +1	1 +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3 0	2 0	1 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3 −1	2 −1	1 −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3 −2	2 −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3 −3
−2 −1	1

1×1

m_1 m_2	2 +2
+1 +1	1

m_1 m_2	2 +1	1 +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2 0	1 0	0 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2 −1	1 −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2 −2
−1 −1	1

3/2×1

m_1 m_2	5/2 +5/2
+3/2 +1	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2 −3/2	3/2 −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2 −5/2
−3/2 −1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

3/2×3/2

m_1 m_2	3 +3
+3/2 +3/2	1

m_1 m_2	3 +2	2 +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	3 +1	2 +1	1 +1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	3 0	2 0	1 0	0 0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

m_1 m_2	3 −1	2 −1	1 −1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

m_1 m_2	3 −2	2 −2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

m_1 m_2	3 −3
−3/2 −3/2	1

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1}_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

m_1 m_2	7/2 +7/2
+2 +3/2	1

m_1 m_2	7/2 +5/2	5/2 +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	7/2 +3/2	5/2 +3/2	3/2 +3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

m_1 m_2	7/2 +1/2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

m_1 m_2	7/2 −1/2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

m_1 m_2	7/2 −3/2	5/2 −3/2	3/2 −3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

m_1 m_2	7/2 −5/2	5/2 −5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

m_1 m_2	7/2 −7/2
−2 −3/2	1

2×2

m_1 m_2	4 +4
+2 +2	1

m_1 m_2	4 +3	3 +3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

m_1 m_2	4 +2	3 +2	2 +2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

m_1 m_2	4 +1	3 +1	2 +1	1 +1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

m_1 m_2	4 0	3 0	2 0	1 0	0 0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

m_1 m_2	4 −1	3 −1	2 −1	1 −1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

m_1 m_2	4 −2	3 −2	2 −2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

m_1 m_2	4 −3	3 −3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

m_1 m_2	4 −4
−2 −2	1

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^{\,2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 34.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.