

Intro to Modern Physics
from the context of the course
PHY 215: Thermodynamics and Modern Physics

Kaedon Cleland-Host

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Chapter 1

Standard Units

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed below:

Type	Unit	Definition
Length	Meter(m)	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram(kg)	Defined by fixing the Planck's constant $h = 6.62607015 \times 10^{-34} kg \cdot m^2 s^{-1}$
Time	Second(s)	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere(A)	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin(K)	Defined by fixing the value of the Boltzmann constant k to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed below:

Prefix	Symbol	Definition
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Chapter 2

Electricity and Magnetism

Law 2.0.1. Maxwells Equations the general laws of electricity and magnetism.

$$\nabla \times \vec{E} = -\vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{D}$$

$$\nabla \cdot \vec{D} = S$$

$$\nabla \cdot \vec{B} = 0$$

Chapter 3

Thermodynamics

Definition 3.0.1. Thermal Expansion is the physical expansion or retraction of materials under temperature changes. Where α = coefficient of linear expansion.

Linear Thermal Expansion:

$$L = L_0(1 + \alpha\Delta T)$$

Volumetric Thermal Expansion:

$$V = V_0(1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3)$$

$$V \approx V_0(1 + 3\alpha\Delta T)$$

Law 3.0.1. Young's Modulus is the constant for the relationship between force applied and a change in length.

$$E = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Where E is the young's modulus, F is the force applied, A is the area over which the force is applied, ΔL is the change in length, and L is the original length.

Law 3.0.2. Heat Capacity Relation is the relationship between thermal energy in materials and their temperature.

$$E = qm\Delta T$$

Where q is the specific heat of the material, m is mass, ΔT is the change in temperature, and E is the amount of energy required or released.

Law 3.0.3. Heat of Phase Transitions - is the energy required/released to break/form inter-molecular bonds that influence the phases of matter.

$$E = L_f m$$

Where L_f is the heat of fusion, m is the mass, and E is the energy required.

Definition 3.0.2. Conduction is the rate of heat energy transfer through a material.

$$E = k \frac{A}{L} \Delta T t$$

Where E is the energy transferred, A is area, L is the length, ΔT is the temperature difference, and t is time.

Law 3.0.4. Unified Gas Law models the behavior of a fixed amount of gas.

$$\frac{PV}{T} = \frac{PV}{T}$$

Where P is pressure, V is volume, and T is temperature.

Law 3.0.5. Ideal Gas Law models the behavior of an ideal gas in a closed container.

$$PV = nRT$$

Where P is pressure, V is volume, n is moles of gas in the material, R is the ideal gas constant $8.31446261815324 \text{ m}^3 \text{ Pa K}^{-1} \text{ mol}^{-1}$, T is temperature.

Remark. To convert from number of particles to moles use the following formula.

$$N = n * 6.022 * 10^{23}$$

Definition 3.0.3. The standard temperature and pressure is a particular temperature and pressure that is used for situations near sea level.

- $T = 273\text{K}$
- $P = 1 \text{ atm} = 101300 \text{ Pa}$
- $V = 22.4 \text{ L} = 0.0224 \text{ m}^3$

Law 3.0.6. The Boltzmann Distribution describes how the velocities of particles at a particular temperature behave.

$$P(v) = Cv^2 e^{-v^2}$$

$$v_p = \sqrt{2} \sqrt{\frac{RT}{M}}$$

$$v_{avg} = \sqrt{\frac{8}{\pi}} \sqrt{\frac{RT}{M}}$$

$$v_{rms} = \sqrt{3} \sqrt{\frac{RT}{M}}$$

Law 3.0.7. The Internal Energy stored by the temperature of a system is modeled by the following formula

$$U = \frac{f}{2} nRT$$

Where f is the degrees of freedom: 3 for monoatomic gasses, 5 for linear molecules, and 6 for asymmetric molecules.

3.1 The Laws of Thermodynamics

Law 3.1.1. The **First Law of Thermodynamics** states that the change in internal energy is given by the sum of change in heat energy times that work done on the system.

$$\Delta U = Q + W$$

Work done to the gas is modeled by the following formula:

$$W = - \int P dV$$

Example. For **Isothermal** processes the temperature is constant so pressure follows $P = \frac{nRT}{V}$ so we can calculate the work done on the system:

$$W = - \int \frac{nRT}{V} dV = -nRT \ln \frac{V_f}{V_i}$$

And from the first law of thermodynamics we can calculate the heat absorbed by the system:

$$Q = -W = nRT \ln \frac{V_f}{V_i}$$

Example. For **Isoobaric** processes the pressure is constant so we can calculate the work done on the system:

$$W = - \int P_0 dV = -P_0 \Delta V$$

And from the first law of thermodynamics we can calculate the heat absorbed by the system:

$$Q = \Delta U - W = \frac{f}{2} nR \Delta T + P_0 \Delta V$$

Example. For **Isochoric(Isovolumetric)** processes the volume is constant so we can calculate the work done on the system:

$$W = 0$$

So clearly the change in internal energy is equal to the heat absorbed by the system:

$$Q = \Delta U = \frac{f}{2} nR \Delta T$$

Example. For **Adiabatic** processes the heat transferred is zero so we can calculate the work done on the system:

$$W = \Delta U = \frac{f}{2} nR \Delta T$$

By definition heat absorbed is zero:

$$Q = 0$$

We can also derive the relationship of pressure and volume:

$$P_1 V_1^{\frac{f+2}{f}} = P_2 V_2^{\frac{f+2}{f}}$$

Law 3.1.2. Second Law of Thermodynamics states that the change in heat energy divided by the temperature is less than or equal to the change in entropy:

$$\frac{\Delta Q}{T} \leq \Delta S$$

Remark. For reversible changes the change in entropy is equal to change in heat.

Definition 3.1.1. The **Efficiency** of a heat pump is the work required to run the pump per heat output.

$$E = \frac{W}{Q_{out}}$$

The **Coefficient of Performance** of a refrigerator is the heat input per work required to run the pump.

$$\kappa = \frac{Q_{in}}{W}$$

Definition 3.1.2. Under the second law of thermodynamic the **Carnot Efficiency** and **Carnot Coefficient of Performance** are the maximum possible efficiency and coefficient of performance for any pump at that temperature.

$$E_C = \frac{T_H - T_C}{T_H}$$

$$\kappa_C = \frac{T_C}{T_H - T_C}$$

Chapter 4

Relativity

Theorem 4.0.1. Principle of Relativity states that the laws of physics are the same in all inertial frames. There is no way to detect absolute motion and there is no preferred inertial frame.

Theorem 4.0.2. Constancy of the Speed of Light states that all observers from any inertial frames measure the speed of light as 299792458m/s.

Theorem 4.0.3. Gravitational Equivalence Principle state that inertia mass and gravitational mass are equal.

Definition 4.0.1. Inertia Frames are reference frames where Newtons first law applies.

Law 4.0.1. Lorentz-Fitzgerald's Change of Reference Frame When changing between a reference frame with coordinate x, y, z, t to a reference frame with a relative velocity v in the x direction and coordinates x', y', z', t' the following equations apply:

$$\begin{aligned}x' &= \gamma(x - \beta ct) \\y' &= y \\z' &= z \\t' &= \gamma(t - \beta \frac{x}{c})\end{aligned}$$

with β and γ defined as

$$\begin{aligned}\beta &= \frac{v}{c} \\ \gamma &= \frac{1}{\sqrt{1 - \beta^2}}\end{aligned}$$

Remark. From this law we can derive an understanding of time dilation and length contraction:

$$\begin{aligned}T' &= \frac{T}{\sqrt{1 - \beta^2}} \\ L' &= L\sqrt{1 - \beta^2}\end{aligned}$$

We can also derive an equation for converting velocities to difference reference frames:

$$U' = \frac{v + U}{1 + \frac{vU}{c^2}}$$

Law 4.0.2. Space-time Interval is the the distance than an object travels through space-time.

$$(\Delta s)^2 = (\Delta r)^2 - (c\Delta t)^2 = (x, y, z, ict)$$

The space-time interval will be the same independent to the reference frame.

Definition 4.0.2. Relativistic Doppler Effect is the effect observed from moving sources of waves. Let f_0 be the original frequency, f be the observed frequency, v_{\parallel} be the component of the relative velocity parallel to the observer, and v_{\perp} be the component of the relative velocity away from the observer.

$$f = f_0 \sqrt{1 - \frac{v_{\parallel}^2}{c^2}} \frac{\sqrt{1 - \frac{v_{\perp}^2}{c^2}}}{\sqrt{1 + \frac{v_{\perp}^2}{c^2}}}$$

Definition 4.0.3. Relativistic Momentum the momentum of an object according to relativity.

$$\rho = \gamma m \vec{v} = \frac{m \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Definition 4.0.4. Relativistic Energy is the energy of objects according to relativity

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Definition 4.0.5. Relativistic Energy and Momentum Relations the following equation describes how momentum and energy are related

$$\begin{aligned}E^2 &= \rho^2 c^2 + (mc^2)^2 \\ \frac{\rho}{E} &= \frac{\beta}{c}\end{aligned}$$

Definition 4.0.6. Relativistic Mass is the mass of objects according to relativity

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Chapter 5

Quantum Physics

5.1 Introduction

5.1.1 Optical Spectroscopy

Definition 5.1.1. Hydrogen Spectrum: J. Rydberg is a model of the emission lines for hydrogen, where n is the final state of the electron, $k > n$ is the initial state ($n, k \in \{1, 2, 3, 4, 5\}$) and $R_H = 1.096776 * 10^7 \text{m}^{-1}$.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{k^2} \right)$$

5.1.2 Black body Radiation

Law 5.1.1. Stefan-Boltzmann Law models the power of thermal radiation released by an object.

$$P = \sigma eAT^4$$

Where σ is the Stefan-Boltzmann constant ($5.6703 * 10^{-8} \text{W/m}^2\text{K}^4$), e is the emissivity, A is the area, and T is the temperature in Kelvin.

Law 5.1.2. Wien's displacement Law models peak wavelength of light emitted by an object as a specific temperature.

$$\lambda_{max} = b/T$$

Where λ_{max} is the wavelength of light at the peak thermal radiation, b is wien's constant ($2.898 * 10^{-3}$), and T is temperature in Kelvin.

Law 5.1.3. The Planck Distribution models the derivative of power output for a black body with frequency or wavelength of radiated light. It was created under the assumption that light has quantized units.

$$I_\lambda = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$
$$I_f = \frac{2\pi h}{c^2} \cdot \frac{f^3}{e^{\frac{hf}{k_B T}} - 1}$$

Law 5.1.4. Planck's constant is the coefficient that relates the frequency of a photon to its energy, recall that this number is one of our fundamentally defined constants as $h = 6.62607015 * 10^{-34} \text{kg m}^2 \text{s}^{-1}$.

$$E_{quanta} = hf$$

Law 5.1.5. The Law of Particles in perpendicular Magnetic and Electric fields is derived from Maxwell's Equations. It described the charge mass ratio of a particle that is deflected at a particular angle in perpendicular Magnetic and Electric fields. There are two possible configurations: a velocity selector where the magnetic and electric field are equal or a spectrometer which separates particles based on mass-charge ratio.

$$\frac{q}{m} = \frac{E \tan \theta}{B^2 l}$$

$$\frac{R}{B} = v_0$$

Law 5.1.6. Photoelectric Effect is the effect observed by shining light on a voltage gap in a vacuum. The energy of one photon is used to accelerate one electron. Let ϕ be the energy required to liberate one electron (work function) from the cathode. We can derive the following equations:

$$hf = \phi + \frac{1}{2}mv^2$$

$$hf = \phi + ev_0$$

Law 5.1.7. Wilhelm Conrad Rontgen discovered the production of x-rays through firing electrons at an cathode in a vacuum. He developed the **Duane-Hunt rule** while models the minimum wavelength photon or highest energy photon that can be produced with this method.

$$\lambda_{min} = \frac{hc}{ev_0}$$

Law 5.1.8. The Compton Effect a model that predicts how a photon will scatter off an electron. Let ϕ represents the deflected angle of the electron and θ be the deflected angle of the photon.

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos(\theta))$$

5.1.3 Structure of an Atom

Law 5.1.9. Rutherford Scattering is a model for predicting the scattering of an alpha particle off a nucleus derived from Coulomb's law.

$$b = \frac{1}{2} \frac{1}{4\pi\epsilon_0} (z_1 e)(z_2 e) \frac{1}{KE} \cot\left(\frac{\theta}{2}\right)$$

where b is the impact parameter and θ is the angle of scattering.

$$f = \frac{\pi}{4} n t (k z_1 e z_2 z)^2 \frac{1}{KE^2} \cot^2\left(\frac{\theta}{2}\right)$$

where f is the fraction of particles that are scattered with an angle greater than θ , n is the number of atoms per unit volume, and t is the thickness.

Differential form:

$$\frac{N(\theta)}{N_i} = \frac{1}{16} n t (k z_1 e z_2 z)^2 \frac{1}{KE^2} \frac{1}{r^2} \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

where $N(\theta)$ is the number of scattered particles per unit area, n is the number of atoms per unit volume, t is the thickness, and r is the radius of the detector.

5.2 Particle Wave Duality

Law 5.2.1. De Broglie Waves predicts the wavelength of both matter and energy (photons) where h is Planck's constant and λ is the wavelength.

$$hf = pc = p\lambda f$$

$$\frac{hc}{pc} = \frac{h}{p} = \lambda$$

Law 5.2.2. Bohr's condition for angular momentum Is the definition of angular momentum of an electron in an orbital it was necessary for the Bohr model of the atom.

$$L = n \frac{h}{2\pi}$$

Law 5.2.3. Clinton Joseph Davisson Electron scattering models the scattering of electrons in a material with a lattice constant D .

$$D \sin(\phi) = n\lambda$$

Law 5.1.10. The Bohr Model of a Hydrogen Atom is a classical model of an atom. In order to complete the model assumptions about the energy states of electron orbitals, and the angular momentum of a state is quantized.

1. Radii of orbits

$$r_n = a_0 \frac{n^2}{z} = n^2 \frac{\epsilon_0 h^2}{\pi m_e e^2 z}$$

$$a_0 \approx 0.529 \text{ \AA}$$

2. Energies of orbits

$$E_n = -E_0 \frac{z^2}{n^2} = -\frac{m e^4 z^2}{8 h^2 \epsilon_0^2 n^2}$$

$$E_0 \approx 13.6 \text{ eV}$$

3. Velocity of orbits

$$\beta_n = \frac{z}{n} \alpha$$

$$\beta_n = \frac{v_n}{c} = \frac{z}{n} \beta_1 = \frac{e^2}{2 \epsilon_0 h c} \frac{z}{n}$$

$$\alpha \approx \frac{1}{137}$$

5.1.4 X-Ray Diffraction

Law 5.1.11. Bragg's Law is a simple model that predicts the scattering angle of x-rays through a crystal with planes separated by a distance d .

$$2d \sin(\theta) = n\lambda$$

5.2.1 Wave mechanics

Definition 5.2.1. A Wave Function is a periodic function with an amplitude. The following is a simple formalism.

$$E(x, t) = A \sin(kx - \omega t)$$

Where x and t are position and time respectively. k is the wave number, ω is the frequency in rad/s , and A is the amplitude.

Example. Wave Functions in Electrodynamics

Waves: $\vec{E}, \vec{B} \in \mathbb{R}^3$

Intensity: $I \sim \vec{E}^2$

Example. Wave Functions in Quantum Mechanics

Waves: $\psi \in \mathbb{C}$

Probability density: $S \sim |\psi|^2$

Definition 5.2.2. Fourier Transformation is a bijection between that translates periodic functions in none periodic functions. In quantum mechanics it used to translated between position and frequency.

Law 5.2.4. Heisenberg's Uncertainty Principle states that there are conjugate variables that we cannot reduced the uncertainty of both below a ratio.

$$\begin{aligned}\delta x \cdot \delta p_x &\geq \frac{h}{4\pi} \\ \delta y \cdot \delta p_y &\geq \frac{h}{4\pi} \\ \delta z \cdot \delta p_z &\geq \frac{h}{4\pi} \\ \delta t \cdot \delta E &\geq \frac{h}{4\pi} \\ \delta \phi \cdot \delta L &\geq \frac{h}{4\pi}\end{aligned}$$

5.3 Quantum Mechanics

Wave functions must be single values, with finite values where the limit: $\lim_{x \rightarrow \infty} \psi = 0$ and they must be continuously differentiable.

Definition 5.3.1. An Operator is a transformation that extracts a physics property from a wave function.

Theorem 5.3.1. Expectation value can be found for a particular operator by integrating the wave function over all space.

$$\langle \hat{O} \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{O} \Psi = \langle \Psi^*, \hat{O} \Psi \rangle$$

Definition 5.3.2. Schrodinger equation is the wave function written by Schrodinger to model quantum measurements.

$$\hat{E}\Psi(x, t) = \hat{K}\Psi(x, t) + \hat{V}\Psi(x, t)$$

Where \hat{E} is the energy operator, \hat{K} is the kinetic energy operator, and \hat{V} is the potential energy operator.

In one dimension we can derive the operators for position, momentum, and energy.

Example. Momentum Operator

$$\hat{P}\Psi(x, t) = -i\frac{h}{2\pi} \frac{\partial}{\partial x} \Psi(x, t)$$

Example. Kinetic Energy Operator

$$\hat{K}\Psi(x, t) = -\frac{h^2}{8\pi^2 m} \frac{\partial^2}{\partial x^2} \Psi(x, t)$$

Example. Total Energy Operator

$$\hat{E}\Psi(x, t) = i\frac{h}{2\pi} \frac{\partial}{\partial t} \Psi(x, t)$$

Now lets consider an example of a wave function for a one dimensional particle.

Example. Infinite Square Well Wave Function is the one dimensional wave function for a particle in a gap.

$$\Psi(x) = A \sin(kx)$$

$$k = \frac{n\pi}{L}$$

$$A = \sqrt{\frac{2}{L}}$$

Now lets apply the energy operator to find the energy spectrum.

$$E = n^2 \frac{h^2}{8mL^2}$$

This example can also be generalized to 3 dimensions. Now we have:

$$\Psi(\vec{x}) = A \sin(k_1 x_1) \sin(k_2 x_2) \sin(k_3 x_3)$$

Note that each dimension can have a different state. So lets find the energy for this 3 dimensional case.

$$E_{n_1, n_2, n_3} = \frac{h^2}{8m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right)$$

$$E_{n_1, n_2, n_3} = \frac{h^2}{8mL^2} (n_1^2 + n_2^2 + n_3^2)$$

Example. Finite Square Well Wave Function is the one dimensional wave function for a particle outside of a gap. In this example the positentials are non-zero for the regions 1 and 3 and zero for the region 2.

$$\Psi_1(x) = Ae^{\alpha x}$$

$$\Psi_2(x) = Ee^{ikx} + Fe^{-ikx}$$

$$\Psi_3(x) = De^{-\alpha x}$$

Example. Simple Oscillator can be modeled using a wave function.

$$\epsilon = \sqrt{\frac{m\omega}{\hbar}} \sqrt{\frac{k}{\omega\hbar}}$$

$$\Psi(\epsilon) = \frac{1}{\sqrt{2^n n! \sqrt{\pi} x_0}} H_n(\epsilon) e^{-\frac{1}{2}\epsilon^2}$$

$$E_n = \left(\frac{1}{2} + n\right) \frac{h}{2\pi} \omega = \left(\frac{1}{2} + n\right) hf$$

5.3.1 The Hydrogen Atom

Definition 5.3.3. The Schrodinger Hydrogen Atom Model uses the techniques of the Schrodinger equation to model the properties of the hydrogen atom.

- **Radial equation**

$$\frac{1}{r^2} \cdot \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2\mu}{\hbar^2} \left[E - V - \frac{\hbar^2}{2\mu} \cdot \frac{\ell(\ell+1)}{r^2} \right] R = 0$$

- **Polar equation**

$$\frac{1}{\sin(\theta)} \cdot \frac{d}{d\theta} \left(\sin(\theta) \cdot \frac{df}{d\theta} \right) + \left[\ell(\ell+1) - \frac{m_\ell^2}{\sin^2(\theta)} \right] f = 0$$

- **Azimuthal equation**

$$\frac{d^2 g}{d\phi^2} = -m_\ell^2 \cdot g$$

Quantum numbers

- principal: $n = 1, 2, 3, \dots$
- orbital angular momentum: $\ell = 0, 1, 2, \dots, (n-1)$
- magnetic: $m_\ell = -\ell, \dots, -1, 0, 1, \dots, \ell$
- spin: $s = \frac{1}{2}$

Wave function

$$\Psi(r, \theta, \phi) = R(r) \cdot f(\theta) \cdot g(\phi)$$

Total Energy

$$E_n = \frac{-13.6\text{eV}}{n^2}$$

Total Orbital angular momentum

$$L = \sqrt{\ell(\ell+1)} \cdot \hbar$$

$$L_z = m_\ell \cdot \hbar$$

Total Spin

$$S = \sqrt{s(s+1)} \cdot \hbar = \frac{\sqrt{3}}{2} \hbar$$

$$S_z = -\frac{1}{2}, \frac{1}{2}$$

Definition 5.3.4. Magnetic moment is defined for an electron by the momentum crossed with the radius.

$$\vec{\mu} = \frac{-e}{2m} \vec{L}$$

$$\mu_z = \frac{-e\hbar}{2m} m_\ell$$

Bohr magneton:

$$\mu_B = \frac{e\hbar}{2m}$$

Definition 5.3.5. Zeeman Effect is the observed effect where applying a magnetic field to an atom splits a state into its magnetic states. Additionally it is possible for the internal magnetic field from the electrons spin. The energy difference between aligned spins and misaligned spins. This is known as the Hyperfine transition.

Chapter 6

Nuclear Physics

Law 6.0.1. Nuclear Radius Approximation is a simple law that can approximate the radius of a nucleus given the number of nucleons in the nucleus.

$$R = r_0 \cdot A^{1/3}; r_0 = 1.2\text{fm}$$

6.0.1 Radioactivity

Some nuclei can spontaneously decay into other nuclei releasing energy. This reaction is based on quantum mechanical probability so it is modeled by exponential decay.

Law 6.0.2. Exponential Radioactive Decay if λ is the probability that a nuclei will decay after one unit of time and N is the current number of remaining nuclei that haven't decayed then we can use differential equations to derive the following equations.

$$\frac{dN}{dt} = \lambda \cdot N$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-\frac{t}{\tau}} = N_0 2^{-\frac{t}{T_{1/2}}}$$

$$\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}}$$

Definition 6.0.1. Binding energy is the energy required to completely separate an arrangement of nuclear particles. Some arrangements have more binding energy and some particles have less binding energy. This determines how stable a particular arrangement is. The isotope Fe-56 has the most binding energy per atomic weight so it is the final fusion product that is no more energy can be gained from nuclear reactions. This binding energy is exactly the difference in mass of the hold nucleus compared to it's individual components.

$$B({}_Z^A X_N) = [Nm_n + Zm_p - M({}_Z^A X_N)]c^2$$

Law 6.0.3. The von Weizsäcker formula models the binding energy of a nucleus.

$$B({}_Z^A X_N) = a_v A - a_A A^{2/3} - \frac{0.72Z(Z-1)}{A^{1/3}} - a_s \frac{(N-Z)^2}{A} + \delta$$

$$a_v = 15.8\text{MeV}$$

$$a_A = 18.3\text{MeV}$$

$$a_s = 23.2\text{MeV}$$

$$\delta = \begin{cases} 33\text{MeV} & \text{even-even} \\ 0 & \text{even-odd, odd-even} \\ -33\text{MeV} & \text{odd-odd} \end{cases}$$

For any nuclear reactions the energy gain or loss of that reaction is determined by the difference in binding energies between the initial nuclei and the final nuclei.

Definition 6.0.2. Alpha Decay is the nuclear reaction that releases a Helium-4 nucleus you can then calculate its disintegration energy as using the following equation

$$Q = [M({}_Z^A X_N) - M({}_{Z-2}^{A-4} X_{N-2}) - M({}_2^4 \text{He}_2)]c^2$$

Definition 6.0.3. Beta Decay is the nuclear reaction that releases an electron or positron you can then calculate its disintegration energy as using the following equation

$$Q = [M({}_Z^A X_N) - M({}_{Z-1}^{A-4} X_N)]c^2$$

$$Q = [M({}_Z^A X_N) - M({}_{Z+1}^{A-4} X_N)]c^2$$