SRF Reference Sheet

Boltzmann's constant $k_B = 1.380649 \times 10^{-23} \mathrm{m}^2\,\mathrm{s}^{-2}\,\mathrm{K}^{-1}$ **Vacuum permeability** $\mu_0 = 1.25663706212 \mathrm{N}\,\mathrm{A}^{-2}$
Planck constant $h = 6.62607015 * 10^{-34} \mathrm{J}\,\mathrm{Hz}^{-1}$
Electron charge $e = 1.60217663 \times 10^{-19} \mathrm{C}$
Electron mass $m_e = 9.1093837 \times 10^{-31} \mathrm{kg}$

Cavity Characteristics

Power Dissipated Per Unit Length

$$\frac{P}{L} = \frac{E_{acc}^2}{\frac{r_a}{Q_0} Q_0}$$

Accelerating Electric Field

The electric field responsible for acceleration E_{acc} is defined in terms of the voltage difference across the cavity V_c divided by the length of the cavity d.

$$E_{acc} = \frac{V_c}{d}$$

The voltage difference V_c can be calculated from the electric field in the center of the cavity and the resonance frequency ω_0.

$$V_c = \left| \int_0^d E_z(z) e^{i\omega_0 z/c} dz \right|$$

Quality Factor

The quality factor Q_0 is the ratio of energy stored to power dissipated in the cavity walls defined in terms of the resonance frequency ω_0, the total energy in the cavity U, and the dissipated power P_c.

$$Q_0 = \frac{\omega_0 U}{P_c}$$

The total energy in the cavity U can be calculated from the electric field E or the magnetic field H.

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv = \frac{1}{2}\epsilon_0 \int_V |\mathbf{E}|^2 dv$$

The power dissipated P_c can be calculated from the magnetic field H and the surface resistance R_s using the following equations.

$$\frac{dP_c}{ds} = \frac{1}{2} R_s |\mathbf{H}|^2$$

$$P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 ds$$

Geometry Factor

The geometry factor G is the component of the quality factor determined by the geometry of the cavity.

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

The quality factor can be written in terms of the geometry factor.

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds} = \frac{G}{R_s}$$

Pill-Box Cavity

Consider a Pill Box cavity of length d and radius R. The solutions The lowest frequency node is

$$E_z = E_0 J_0 \left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

$$H_\phi = -i\frac{E_0}{\eta} \left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

The resonance frequency of such a cavity is

$$\omega_{010} = \frac{2.405c}{R}$$

Here the nomenclature ω_{010} is realted to the number of sign changes E_z in the ϕ, ρ, z directions respectively (cylindrical coordinates). For the TM_{010} pill-box mode, $E_{acc} = 2E_0/\pi$ and the peak fields are

$$E_{pk} = E_0, H_{pk} = \frac{E_0}{\eta} J_1(1.84) = \frac{E_0}{647\Omega}$$

If $d = 10cm, R = 7.65cm, \omega_0 = 1.5GHz, V_c = 1MV$ then

$$U = E_0^2 \frac{\pi\epsilon_0}{2} J_1^2(2.405) dR^2 = 0.54J$$

$$P_c = \frac{\omega U}{Q_0} = 0.4W$$

Shunt Impedance

The shunt impedance R_a is an important quantity used to characterize losses.

$$R_a = \frac{V_c^2}{P_c}$$

Another definition for shunt impedance r_a is

$$r_a = \frac{V_c^2}{P_c'}$$

Conductivity

Electrical conductivity denoted σ is defined

$$\mathbf{j} = \sigma\mathbf{E}, \sigma = \frac{ne^2\tau}{m_e}$$

where $\mathbf{j}$ is the current, $\mathbf{E}$ is the electric field, n is the number of electrons per cubic centimeter, and τ is the average time between collisions.

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

The average time between collisions deceases with higher temperature so conductivity increases with decreasing temperature.
Thermal Conductivity denoted κ is defined

$$j_p = -\kappa \frac{dT}{dx}$$

due the pauli exclusion principle the relationship between thermal conductivity (from electrons) and electric conductivity is

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2 T$$

Note there is also thermal conductivity from phonons but the contribution is insignificant at low temperatures. The equation for the relationship between thermal conductivity (from electrons) and electric conductivity is

$$\frac{\kappa}{\sigma} = \frac{mv^2 C_v}{3ne^2}$$

The specific heat equations are

$$C_v = \frac{\pi^2}{2} \left(\frac{k_B T}{\varepsilon_F}\right) n k_B$$

$$C_v = \frac{\pi^2}{2} k_B^2 T D(\varepsilon)$$

Superconducting specific heat (Δ is the superconducting gap)

$$C_s \propto e^{-\frac{\Delta}{k_B T}}$$

Cooper pair coherence length

$$\xi_0 = \frac{\hbar v_F}{k_B T_c} = \frac{\hbar v_F}{\Delta}$$

BCS theory band gap approximation

$$\frac{\Delta(T)}{\Delta(0)} = \left[\cos\left(\frac{\pi t^2}{2}\right)\right]^{1/2}$$

Fraction of unpaired electrons

$$n_{normal} \propto e^{-\frac{\Delta}{k_B T}}$$

Critical current (maximum superconducting current)

$$J_c = \frac{2en\Delta}{m_e v_F}$$

Surface rf resistance due to a small amount of surface electric field penetration

$$R_s = A_s \omega^2 e^{-\frac{\Delta(0)}{k_B T}}$$

This is why we cool to 2K. Thermal conductivity is related to RRR

$$RRR = 4k_s (W/meterK)$$

Maximum Surface Fields

Page 92 with γ defined for equation (3.26)

$$\frac{\mu_0 H_c^2}{2} = 0.236\gamma T_c^2$$

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Type I superconductors have a positive surface energy that prevents magnetic fields from entering until the magnetic field is strong enough to completely break down superconductivity. Type II superconductors have a negative surface energy so there is a smaller critical magnetic field H_{c1} such that the magnetic fields penetrate the superconductor. The Ginzbur-Landau parameter distinguishes between type I and type II parameters defined as

$$\kappa_{GL} = \frac{\lambda_L}{\xi_0}$$

where ξ_0 is the coherence distance and λ_L is the magnetic penetration distance.

$$k_{GL} < \frac{1}{\sqrt{2}} \text{Type I}, \quad k_{GL} > \frac{1}{\sqrt{2}} \text{Type II}$$

Φ_0 is the flux quantum defined $\Phi_0 = \frac{h}{2e}$. Flux quantum is related to the critical magnetic fields H_{c1} and H_{c2} (5.12):

$$H_{c2} = \frac{\Phi_0}{2\pi\mu_0\xi_0^2}, \quad H_c = \frac{H_{c2}}{\sqrt{2}\kappa_{GL}}$$

$$H_{c1} \propto \frac{H_c}{\sqrt{2}\kappa_{GL}} \ln(\kappa_{GL}) = \frac{\Phi_0}{4\pi\mu_0\lambda_L^2} \ln(\kappa_{GL})$$

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